THE IMPORTANCE OF LIMIT SOLUTIONS & TEMPORAL AND SPATIAL SCALES IN THE TEACHING OF TRANSPORT PHENOMENA

La importancia de soluciones limite & escalas temporales y espaciales en la enseñanza del fenómeno del transporte

**SÁVIO LEANDRO BERTOLI,[A] JUSCELINO DE ALMEIDA JÚNIOR,[B]
JULIANO DE ALMEIDA,[C] ALINE CRISTINA LOVATEL,[D] JACKSON R. ELEOTERIO,[E]
SIMONE LEAL SCHWERTL,[F] PAULO ROBERTO BRANDT[G]**

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Abstract

In the engineering courses the field of Transport Phenomena is of significant importance and it is in several disciplines relating to Fluid Mechanics, Heat and Mass Transfer. In these disciplines, problems involving these phenomena are mathematically formulated and analytical solutions are obtained whenever possible. The aim of this paper is to emphasize the possibility of extending aspects of the teaching-learning in this area by a method based on time scales and limit solutions. Thus, aspects relative to the phenomenology naturally arise during the definition of the scales and / or by determining the limit solutions. Aspects concerning the phenomenology of the limit problems are easily incorporated into the proposed development, which contributes significantly to the understanding of physics inherent in the mathematical modeling of each limiting case studied. Finally the study aims to disseminate the use of the limit solutions and of the time scales in the general fields of engineering.

Keywords: Transport phenomena, limit solutions and limit operations, spatial and temporal scales, analytical solutions.

Resumen

En los cursos de ingeniería el campo de los fenómenos de transporte es de gran importancia y se encuentra en varias disciplinas relacionadas con la mecánica de fluidos, transferencia de calor y masa. En estas disciplinas, los problemas que implican estos fenómenos son matematicamente formulados y las soluciones analíticas son obtenidas como sea posible. El objetivo de este trabajo es resaltar la posibilidad de ampliar los aspectos de la enseñanza-aprendizaje en esta área mediante un método basado en escalas de tiempo y soluciones límite. Por lo tanto, los aspectos relativos a la

[A] Professor of Regional University of Blumenau. Department of Chemical Engineering. Experience in the field of Mathematical Physics with emphasis on Heat Transfer, Thermal Processes and Thermodynamic, specifically in the following topics: Heat Transfer in multi-particulate systems, Analytical Solutions in Transport Phenomena, Flows in Stokes regime, Advanced Oxidation Processes. Contact: savio@furb.br / +55 (47) 3221-6051.

[B] Researcher and master degree in Chemical Engineering at the Regional University of Blumenau. Experience in Analytical Solutions and Semi-Analytical Solutions in Transport Phenomena.

[C] Currently enrolled in the master degree program in Chemical Engineering at the Regional University of Blumenau.

[D] Researcher and graduated in Chemical Engineering from the Regional University of Blumenau.

[E] Professor of the Regional University of Blumenau. Department of Forestry Engineering. Experience in the area of Wood Science and Technology with emphasis in Wood-Water relationship.

[F] Professor of the University Foundation of Blumenau. Departament of Mathematics. Experience in Differential and Integral Calculus, Numerical Calculus, Algebra and Geometry.

[G] Professor of the Regional University of Blumenau. Department of Eletrical Engineering. Experience in Electrical Engineering with emphasis in Electronics and Microelectronics, mainly in the following areas: telecommunications, broadcasting, alternative energy, energy co-generation, power generation, energy and electromagnetism.

fenomenología surgen naturalmente durante la definición de las escalas y/o mediante la determinación de las soluciones límite. Aspectos relativos a la fenomenología de los problemas de límite se incorporan fácilmente en el desarrollo propuesto, lo que contribuye significativamente a la comprensión de la física inherentes a la modelización matemática de cada caso límite estudiado. Por último, el estudio tiene como objetivo difundir el uso de las soluciones de límite y de las escalas de tiempo en los campos generales de ingeniería.

Palabras clave: Fenómeno del transporte, soluciones límite y las operaciones limite, las escalas espaciales y temporales, las soluciones analíticas.

I. Introducción

The transport phenomena comprises a broad area of science and integrate the curriculum of various courses of engineering such as: Chemical Engineering, Mechanical Engineering, Electrical Engineering, Industrial Engineering, etc. The formulation of problems in this area follows usually the Eulerian approach [1] involving mathematical models based on differential equations that express the dependency of variables such as temperature, concentration and speed in the variables spatial and/or temporal variable. Furthermore, analytical solutions of different problems in this area are liable to be reduced to solutions of particular cases by a method based on limit operations used initially by[2] and detailed in [3]. The method used [2], [3], [4] provides that from a general solution to a given problem, limiting solutions are obtained and it is important to note that in many cases this is not a trivial task [2], [3]. Thus, the limiting solutions are obtained without the need for resolving again the equations of the reduced model. Simultaneously, this allows a deepening of physics at the problem through the establishment of limit conditions and analysis of the limit solutions obtained. Another important aspect along with the limit solutions that has been systematically ignored in different undergraduate textbooks of Transport Phenomena is the question of definition of scales in the problem formulation. Once properly established, the spatial and temporal scales assist the understanding of the involved phenomena and may also facilitate aspects of teaching and learning in the field of Transport Phenomena [5].

This work demonstrates that the use of scales and limit solutions allows to expand the teaching aspects of Transport Phenomena, particularly in the understanding at the phenomenology [5], [6]. In this sense, the paper is organized in the following way: Section 2 presents as a case study «Heat Conduction in a Flat Plate» and the solutions of the literature for the chosen models are presented. In the Section 3 the scales of time and space for the case study are defined. Based on these scales, the physical meaning of dimensionless groups present in the phenomenon are given. In section 4, taking as basis the method of obtaining limit solutions detailed in [3], limit solutions are obtained and the results interpreted physically. Finally, in section 5, we report the findings, and, issues related to education in engineering are emphasized.

II. Case study - heat conduction on a flat plate

To illustrate the use of limit solutions and time scales in engineering teaching, it is proposed as a case study the «Heat Conduction in a Flat Plate.» The assumptions, the governing equation and the initial condition for this problem are presented in the following.

Assumptions: The board is rectangular and infinite in length and width; the initial temperature of the plate is uniform; heat transfer occurs by both sides; physical properties are uniform and constant.

Governing equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (1)$$

Where:

T is the temperature, (K);
α is the thermal diffusivity of the plate, (m2/s);
t is the time, (s);
x is the position relative to the center of the plate, (m).

Initial Condition:

$$t = 0,\ -L \leq x \leq +L:\ T = T_0 \quad (2)$$

Where,
T_0 is the initial temperature of the plate, (K);
L is the half-thickness of the plate, (m).

The governing equation is known as the diffusion equation. Determining solutions to this equation means obtaining functions that relate the temperature as a function of position and time. These solutions are achieved from the establishment of boundary conditions and knowledge of the physical meaning of the boundary conditions imposed [7]. In this case study boundary conditions of the third and first type will be chosen with the models called "Model I" and "Model II", respectively. These boundary conditions are presented respectively below.

Model I -Boundary Conditions:

$$x = -L, t > 0 : \ \alpha \frac{\partial T}{\partial x} = h_1(T - T_s) \tag{3}$$

$$x = +L, t > 0 : \ -\alpha \frac{\partial T}{\partial x} = h_2(T - T_s) \tag{4}$$

Where:

h is the heat transfer coefficient, (m/s);
T_s in this model is the temperature of the heating/cooling fluid, (K).

Model II - Boundary Conditions:

$$x = -L, t > 0 : T = T_s \tag{5}$$

$$x = +L, t > 0 : T = T_s \tag{6}$$

Where:

T_s in this model is the surface temperature of the plate, (K).
Defining now the following dimensionless variables:

$$x^* \equiv \frac{x}{L} \tag{7}$$

$$t^* \equiv \frac{t\alpha}{L^2} = \text{Fo} \tag{8}$$

$$T^* \equiv \frac{T(x,t) - T_s}{T_0 - T_s} \tag{9}$$

where "Fo" is the Fourier Number, the Eqs. (1)-(4) are rewritten in the form:

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial x^{*2}} \tag{10}$$

$$t^* = 0, -1 \le x^* \le +1 : T^* = 1 \tag{11}$$

$$x^* = -1, t^* > 0 : \frac{\partial T^*}{\partial x^*} = BiT^* \tag{12}$$

$$x^* = +1, t^* > 0 : \frac{\partial T^*}{\partial x^*} = -BiT^* \tag{13}$$

Where "Bi" is the Biot Number, expressed as:

$$Bi = \frac{hL}{\alpha} \tag{14}$$

The Biot number represents the ratio between the internal thermal resistance to conduction in a solid and the surface resistance to convection heat transfer [8]. In equations (12) and (13) it is also assumed that:

$$h_1 = h_2 = h \tag{15}$$

In terms of dimensionless variables, the boundary conditions of the Model II, Equations (5) and (6) are rewritten as:

$$t^* > 0, x^* = -1 : T^* = 0 \tag{16}$$
$$t^* > 0, x^* = +1 : T^* = 0 \tag{17}$$

The solution of the Model I (Eqs. 10-13) can be found for instance in [9] or [10]:

$$T^*(x^*, t^*) = 2Bi \sum_{n=1}^{\infty} \frac{\cos(\beta_n x^*) \exp(-\beta_n^2 t^*)}{(\beta_n^2 + Bi^2 + Bi)\cos(\beta_n)} \tag{18}$$

Where the "$\beta_n s$" are the positive roots of:

$$\beta \tan(\beta) = Bi \tag{19}$$

For the Model II (Eqs. 10, 11, 16, 17), the solution can be found, for example, in [11] or [12]:

$$T^*(x^*, t^*) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} cos\left[\frac{(2n+1)}{2}\pi x^*\right] \times exp\left\{-\frac{(2n+1)^2\pi^2}{4} t^*\right\} \tag{20}$$

III. Spatial and temporal scales

In order to emphasize the importance of scales in the physical understanding of problems in Transport Phenomena and therefore in Engineering teaching, the following scales are defined for the case study:

(a) Spatial scale:

L_x=L= Spatial scale in the x direction

(b) Time Scales:

$t_\alpha = \frac{L^2}{\alpha} =$ Time scale for propagation of the conductive effects to the distance L_x inside the plate.

$t_h = \frac{L}{h} =$ Time scale for the heating or cooling of the plate surface by convection between the fluid and the plate.

As mentioned, setting up the scales, the physical meaning of dimensionless groups present on a particular problem arise naturally. Thus,

$$\mathrm{Bi} = \frac{t_\alpha}{t_h} = \frac{L^2/D}{L/h} = \frac{hL}{D} \tag{21}$$

Bi= time scale for propagation of the conductive effects to a distance L within the plate/time scale for the heating or cooling of the plate surface by convection.

$$Fo = t^* = \frac{t}{t_\alpha} \tag{22}$$

Fo= time/time scale for propagation of the conductive effects to a distance L within the plate.

Note that the use of scales allows an alternative interpretation for the Biot number, in relation to usual interpretation [8], based in the concept of thermal resistance.

The solutions for Models I and II (Equations 18-20) are rewritten in terms of the previous scales as:

Model I:

$$T^*(x^*,t^*) = 2\left(\frac{t_\alpha}{t_h}\right)\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L_x)\exp\left(-\beta_n^2 t/t_\alpha\right)}{\left[\beta_n^2+\left(\frac{t_\alpha}{t_h}\right)^2+\left(\frac{t_\alpha}{t_h}\right)\right]\cos(\beta_n)} \tag{23}$$

Where the "$\beta_n s$" are the positive roots of:

$$\beta\tan(\beta) = t_\alpha/t_h \tag{24}$$

Model II:

$$T^*(x^*,t^*) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}\cos\left[\frac{(2n+1)\pi}{2}\left(\frac{x}{L_x}\right)\right] \times \exp\left\{-\frac{(2n+1)^2\pi^2}{4}\left(\frac{t}{t_\alpha}\right)\right\} \tag{25}$$

In the solution represented by the Eq. (25), the scales need to be finite and nonzero. For example, it is noted by Eq. (22), that t_α needs to be finite and non-zero to keep as a scale in the solution. If $t_\alpha = 0$ the conductive phenomenon occurs instantaneously; and if $t_\alpha \to \infty$ the phenomenon of heat conduction to a distance L inside the plate does not occur in a finite time. So, in both of these cases, t_α cannot serve to scale t in the solution.

IV. Determination of the limit solutions

In this section, based on the method for determining limiting solutions developed in [2] and detailed in [3] and considering the scales defined in section 3, the limiting solutions of Model I (section 2) are determined. In the process, the physical aspects of the problem are explored which facilitates the understanding and teaching of phenomenology.

A. Limit Solution for the Model I when Bi → ∞

In this case, as can be interpreted from the Biot number based on scales, (Eq. 21), there are two possibilities for Bi $\to \infty$: (i) t_α = finite and $t_h \to 0$, or (ii) $t_\alpha \to \infty$ and t_h = finite or zero; but under section 3, only (i) allows t_α be maintained as a scale. Then, this is the first limiting case considered.

1) *Bi* $\to \infty$ *with* $t_\alpha = finite\ and\ t_h \to 0$

For Bi $\to \infty$, the root´s equation (19) takes the form:

$$\tan(\beta) = \infty \Rightarrow \beta_n = \pi\left(n-\frac{1}{2}\right); n = 1, 2, \ldots \tag{26}$$

Note that when applying $\lim_{\mathrm{Bi}\to\infty} T^*(x^*, t^*)$ in Eq. (18), indeterminacies of the type (∞/∞) appears. The "algebraic reconstruction" and the use of the root´s equation, is the procedure to raise this indeterminacy as described in [3]. Starting with the "algebraic reconstruction" of Equation (18), we have:

$$T^*(x^*,t^*) = 2\sum_{n=1}^{\infty} \frac{\cos(\beta_n x^*)\exp(-\beta_n^2 t^*)}{(\beta_n^2 Bi^{-1} + Bi + 1)\cos(\beta_n)} \quad (27)$$

It is observed that even after of the algebraic reconstruction, the form given by Eq. (27) still retains an indetermination of the type (∞ × 0) in the denominator of each term in the series. To raise this indetermination, the root's equation, Eq. (19), is substituted in Eq. (27), obtaining:

$$T^*(x^*,t^*) = 2\sum_{n=1}^{\infty} \frac{\cos(\beta_n x^*)\exp(-\beta_n^2 t^*)}{(\beta_n^2 Bi^{-2} + 1 + Bi^{-1})\beta_n \sin(\beta_n)} \quad (28)$$

Applying the operation $\lim_{Bi\to\infty}$ in the Eq. (28), with the use of Equation (26), results:

$$\lim_{\substack{Bi\to\infty \\ t_h\to 0}} T^*(x^*,t^*) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1} cos\left[\frac{(2n+1)}{2}\pi x^*\right] \times exp\left\{-\frac{(2n+1)^2\pi^2}{4}t^*\right\} \quad (29)$$

It is verified that the limit solution given by Eq. (29) corresponds exactly to the solution of the Model II expressed by Eq. (20). This is the result physically expected, since for Bi → ∞ with t_α = finite and $t_h \to 0$, the heat transfer by convection at the surface of the plate is infinitely faster than conduction heat transfer in its interior, thus making the surface to remain at the constant temperature of the fluid. This situation is equivalent to the boundary conditions of the Model I.

2) Bi → ∞ with $t_\alpha \to \infty$ and t_h = finite or null

In establishing these limit conditions it is found that the Fourier number becomes null; then it should not be used t_α as a scale for t. Thus, the solution is reconstructed to incorporate this change, noting that:

$$\beta_n^2 Fo = \left(\frac{\beta_n}{\sqrt{Bi}}\right)^2 Bi\ Fo = \gamma_n^2\ Bi\ Fo = \gamma_n^2 \frac{t}{t_h}$$

Where was done: $\gamma_n = \frac{\beta_n}{\sqrt{\mathrm{Bi}}}$

In this case, from the root's equation, Eq. (26), we have:

$$\gamma_n\sqrt{Bi} = \pi\left(n-\frac{1}{2}\right); n = 1, 2, ... \quad (30)$$

Where one can conclude that for finite "n", is required that γ_n=0

Rewriting the solution of the Model I in the form given by equation (27) in terms of γ_n, and t_h, we have:

$$T^*(x,t) = 2\sum_{n=1}^{\infty}\frac{\cos(\gamma_n\sqrt{Bi}\ ^x/_L)\exp(-\gamma_n^2\ ^t/_{t_h})}{(\gamma_n^2 + Bi + 1)\cos(\gamma_n\sqrt{Bi})} \quad (31)$$

Substituting Eq. (30) in the last expression and applying the limits for this case, we have:

In $x = \pm L$

$$\lim_{\substack{Bi\to\infty \\ t_\alpha\to\infty}} T^*(\pm L,t) = 2\lim_{\substack{Bi\to\infty \\ t_\alpha\to\infty}}\sum_{n=1}^{\infty}\frac{\cos(\gamma_n\sqrt{Bi}\ ^x/_L)\exp(-\gamma_n^2\ ^t/_{t_h})}{(\gamma_n^2 + Bi + 1)\cos(\gamma_n\sqrt{Bi})} = 0 \quad (32)$$

or,

$$\lim_{\substack{Bi\to\infty \\ t_\alpha\to\infty}} T(\pm L,t) = T_s\ \ for\ t > 0 \quad (33)$$

For $-L < x < +L$, an indetermination of the kind (∞ × 0) arises in the denominator of the series of Eq. (32). In this case, applying L'Hospital, and summing the resulting series, we obtain:

$$\lim_{\substack{Bi\to\infty \\ t_\alpha\to\infty}} T(x,t) = T_0 \quad for\ (-L < x < +L) \quad (34)$$

The results expressed by Eqs. (33) and (34) correspond exactly to those expected when the propagation time of the conductive effects at a distance L is infinitely greater than the time for convective heat transfer on the plate surface; in this situation, for $t > 0$, the surface of the plate is in the temperature of the fluid, while the interior remains at the initial temperature.

B. Limit Solution for the Model I when Bi → 0

In this case, according to the interpretation of the Biot number based on scales (Eq. 21), there are two possibilities for Bi → 0: (i) t_α finite and $t_h \to \infty$; (ii) $t_\alpha \to 0$ and t_h finite; but as mentioned in section 3, only (i) allows that t_α be maintained as the time scale in question. We analyze these cases in the following.

1) Bi $\rightarrow 0$ with t_α finite and $t_h \rightarrow \infty$

For Bi $\rightarrow 0$ the root's equation, Eq. (19) takes the following form:

$$\beta_1 = 0$$

$$\beta \tan(\beta) = 0 \Rightarrow \beta_n = (n-1)\pi; \quad n = 1, 2 \ldots \tag{35}$$

Where $\beta_1=0$ it is a double root.

When applying Bi $\rightarrow 0$ in Eq. (18), all terms of the sum result null, except the first term which is undetermined for "$\beta_1=0$" , then:

$$\lim_{\substack{Bi\to\infty \\ t_h\to\infty}} T^*(x^*, t^*) = \lim_{\substack{Bi\to 0 \\ \beta_1\to 0}} \frac{Bi}{\beta_1^2} \tag{36}$$

The procedure for removal of such indetermination is the use of initial condition [2], [3], [4] which in this case is given by Eq. (11):

$$\lim_{\substack{Bi\to\infty \\ t_h\to\infty}} T^*(x^*, 0) = 1 = \lim_{\substack{Bi\to 0 \\ \beta_1\to 0}} \frac{Bi}{\beta_1^2} \tag{37}$$

Thus,

$$\lim_{\substack{Bi\to 0 \\ t_h\to\infty}} T^*(x^*, t^*) = \lim_{\substack{Bi\to 0 \\ t_h\to\infty}} \frac{T(x,t) - T_s}{T_0 - T_s} = 1 \tag{38}$$

or,

$$\lim_{\substack{Bi\to 0 \\ t_h\to\infty}} T(x,t) = T_0 \tag{39}$$

This result is physically expected since to Bi $\rightarrow 0$ with t_α finite, and $t_h \rightarrow \infty$ it means that heat transfer by convection at the plate surface occurs at a time infinitely greater than the heat conduction inside the plate, and since the initial temperature is uniform, the plate always remains at the same condition.

It is considered hereinafter, the second possibility.

2) Bi $\rightarrow 0$ with $t_\alpha \rightarrow 0$ and $t_h =$ finite or zero.

In establishing these limit conditions the Fourier number becomes infinite; one should then use the scale t_h as a t scale. Thus, the solution it is reconstructed to incorporate this change, observing that:

$$\beta_n^2 Fo = \left(\frac{\beta_n}{\sqrt{Bi}}\right)^2 Bi\, Fo = \gamma_n^2\, Bi\, Fo = \gamma_n^2 \frac{t}{t_h}$$

Where: $\gamma_n = \frac{\beta_n}{\sqrt{\mathrm{Bi}}}$

In parallel, the root´s equation (Eq. 19) is written as $\gamma\sqrt{\mathrm{Bi}}\tan(\gamma\sqrt{\mathrm{Bi}}) = \mathrm{Bi}$, therefore:

$$\lim_{Bi\to 0} \gamma = \lim_{Bi\to 0} \frac{\sqrt{Bi}}{\tan(\gamma\sqrt{Bi})} = \frac{0}{0}$$

Applying L'Hospital obtains a single root: $\gamma_1=1$.

Substituting this result in the solution of Model I expressed by Eq. (31), we obtain:

$$\lim_{\substack{Bi\to 0 \\ t_\alpha\to 0}} T^*(L,t) \tag{40}$$
$$= 2\lim_{\substack{Bi\to 0 \\ t_\alpha\to 0}} \sum_{n=1}^{\infty} \frac{\cos(\gamma_n\sqrt{Bi}\, ^x/_L)\exp\left(-\gamma_n^2\, ^t/_{t_h}\right)}{(\gamma_n^2 + Bi + 1)\cos(\gamma_n\sqrt{Bi})} = e^{-\frac{t}{t_h}}$$
$$= e^{-\frac{ht}{L}}$$

or,

$$\lim_{\substack{Bi\to 0 \\ t_\alpha\to 0}} \frac{T(x,t) - T_s}{T_0 - T_s} = e^{-\frac{ht}{L}} = \frac{T(t) - T_s}{T_0 - T_s} \tag{41}$$

The Eq. (41) corresponds exactly to the solution of a lumped model for heat transfer in a flat plate of thickness "2L". This is the expected physical situation in which case the conduction into the plate is infinitely faster than the convection at the surface.

Conclusions

The combination of techniques for establishing scales and obtain limit solutions, allowed elucidate various aspects of the phenomenology of the case study discussed in section 2. A physical understanding is essential in the teaching of Transport Phenomena, which is part of the "curriculum" of several Engineering courses. Through the development proposed in this work and in a previous work [3] it is expected that the teaching of this matter can be facilitated. The proposed method of "Scales & Limit Solutions" is quite general for use in other areas of Education in Engineering and proved to be very suitable for obtaining solutions of problems of Mathematical Physics [3], [4]. The techniques used in the rising of indeterminacies were also established, which are essentials for obtaining the limit solutions. It is hoped that this work will help in disseminate the mentioned method and the concepts related to it.

Experience teaching in engineering courses allows us to put that this is the case of the concept of limit. The case study presented and the determination of solutions point to another opportunity to develop this concept over the course of engineering, besides emphasizing its relevance.

An interesting feature in obtaining limit solutions is that indeterminacies may be viewed as "mathematical blocks"; their correct identification and removal is the "path" to obtain the solution. Once established the limit values of the parameters, this "path" comes naturally [3].

Finally, it is proposed that the techniques at limit operations used in this article, may under a general perspective, disseminate in solving other problems in the teaching of Engineering, for indeed, there is no restriction about the implementation of the method in other areas.

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